MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Thursday 6 June 2019

Afternoon (Time: 1 hour 30 minutes)

Paper Reference **9FM0/02**

Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P61178A

1/1/1/1/

P 6 1 1 7 8 A 0 1 2 8

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k.

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

a) Let $y = \tanh^{-1}(x)$,

$$\Rightarrow x = \tanh(y) = \frac{\sinh(y)}{\cosh(y)}$$

$$= \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

$$x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \times e^y = \frac{e^{2y} - 1}{e^{2y} + 1}$$

$$x(e^{2y}+1) = e^{2y} - 1$$

$$xe^{2y} + x = e^{2y} - 1$$

$$xe^{2y} - e^{2y} = -x - 1$$

$$e^{2y}(x-1) = -(x+1)$$

$$e^{2y} = \frac{1+x}{1-x}$$

$$2y = \ln\left(\frac{1+x}{1-x}\right)$$

$$y = \tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ (as required)

Since ln( ) only takes +ve arguments,

$$\left.\begin{array}{l} 1+x > 0 \\ 1-x > 0 \end{array}\right\} \Rightarrow -1 < x < 1 \qquad \therefore k = 1$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

b) $$2x = \tanh\left(\ln\left(\sqrt{2-3x}\right)\right)$$

$$\tanh^{-1}(2x) = \ln\left((2-3x)^{1/2}\right)$$

$$\frac{1}{2}\ln\left(\frac{1+2x}{1-2x}\right) = \frac{1}{2}\ln(2-3x)$$

By equating arguments of the natural logs,

$$\Rightarrow \frac{1+2x}{1-2x} = 2-3x$$

$$1+2x = 6x^2-7x+2$$

$$6x^2-9x+1=0$$

$$x = \frac{9 \pm \sqrt{9^2-4\times6\times1}}{2\times6}$$

$$x = \frac{9\pm\sqrt{57}}{12}$$

Since $-1 < x < 1$,

$$x = \frac{9-\sqrt{57}}{12}$$

(Total for Question 1 is 10 marks)

2. The roots of the equation

$$x^3 - 2x^2 + 4x - 5 = 0$$

are p, q and r.

Without solving the equation, find the value of

(i) $\dfrac{2}{p} + \dfrac{2}{q} + \dfrac{2}{r}$

(ii) $(p-4)(q-4)(r-4)$

(iii) $p^3 + q^3 + r^3$

(8)

Using Vieta's Formulae (From 'Roots of Polynomials' Chapter),

$$p+q+r = -\frac{(-2)}{1} = 2$$

$$pq+pr+qr = \frac{4}{1} = 4$$

$$pqr = -\frac{(-5)}{1} = 5$$

(i) $\dfrac{2}{p} + \dfrac{2}{q} + \dfrac{2}{r} = \dfrac{2(pq+pr+qr)}{pqr}$

$= \dfrac{2 \times 4}{5} = \dfrac{8}{5}$

(ii) $(p-4)(q-4)(r-4) = (p-4)(qr - 4q - 4r + 16)$

$= pqr - 4pq - 4pr + 16p - 4qr + 16q + 16r - 64$

$= pqr - 4(pq+pr+qr) + 16(p+q+r) - 64$

$= 5 - 4(4) + 16(2) - 64$

$= -43$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 1 1 7 8 A 0 4 2 8

Question 2 continued

(iii)

$$p^3+q^3+r^3 = (p+q+r)^3 - 3(p+q+r)(pq+qr+pr) + 3pqr$$

$$= (2)^3 - 3(2)(4) + 3(5)$$

$$= -1$$

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 8 marks)

3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)\,dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval $[0, 3]$.

(2)

(a) Let $x = \frac{3}{2}\sinh(u) \Rightarrow dx = \frac{3}{2}\cosh(u)\,du$

$\Rightarrow f(x) = \dfrac{1}{\sqrt{4\left(\frac{3}{2}\sinh u\right)^2 + 9}}$

$= \dfrac{1}{3\sqrt{\sinh^2 u + 1}}$

$= \dfrac{1}{3\cosh u}$

$\therefore \int f(x)\,dx = \int \dfrac{1}{3\cosh u} \times \dfrac{3}{2}\cosh u \; du$

$= \dfrac{1}{2}\int 1\,du$

$= \dfrac{u}{2} + c$

$x = \dfrac{3}{2}\sinh u \Longleftrightarrow u = \sinh^{-1}\left(\dfrac{2}{3}x\right)$

$\Rightarrow \int f(x)\,dx = \dfrac{1}{2}\sinh^{-1}\left(\dfrac{2}{3}x\right) + c$

$A = \dfrac{1}{2}, \quad B = \dfrac{2}{3}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 1 1 7 8 A 0 8 2 8

DO NOT WRITE IN THIS AREA

Question 3 continued

(b) Mean value of $f(x)$ over $[0, 3] = \frac{1}{3-0}\int_0^3 f(x)\,dx$

$$= \frac{1}{3}\left[\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)\right]_0^3$$

$$= \frac{1}{6}\left[\sinh^{-1}(2) - \sinh^{-1}(0)\right]$$

$$= \frac{1}{6}\sinh^{-1}(2)$$

Using logarithmic form of $\sinh^{-1}$ from formula book,

$$\text{arsinh } x = \sinh^{-1} x$$
$$= \ln\left(x + \sqrt{x^2+1}\right)$$

$$\Rightarrow \bar{f}(x) = \frac{1}{6}\ln\left(2+\sqrt{2^2+1}\right)$$
$$= \frac{1}{6}\ln\left(2+\sqrt{5}\right)$$

(Total for Question 3 is 6 marks)

P 6 1 1 7 8 A 0 9 2 8

4. The infinite series C and S are defined by

$$C = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \ldots$$

$$S = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \ldots$$

Given that the series C and S are both convergent,

(a) show that

$$C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}}$$

(4)

(b) Hence show that

$$S = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

(4)

(a) $C + iS = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \ldots$
$+ i\left(\sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \ldots\right)$

$= (\cos\theta + i\sin\theta) + \frac{1}{2}(\cos 5\theta + i\sin 5\theta) + \frac{1}{4}(\cos 9\theta + i\sin 9\theta) + \ldots$

Using Euler's Formula,

$= e^{i\theta} + \frac{1}{2}e^{5i\theta} + \frac{1}{4}e^{9i\theta} + \frac{1}{8}e^{13i\theta} + \ldots$

This is an infinite geometric series, with

First term $= a = e^{i\theta}$
Common ratio $= r = \frac{1}{2}e^{4i\theta}$

$\therefore\ C + iS = \frac{a}{1-r} = \frac{e^{i\theta}}{1 - \frac{1}{2}e^{4i\theta}} = \frac{2e^{i\theta}}{2 - e^{4i\theta}}$ (as required)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 4 continued

(b) Using a similar technique as rationalizing surds, multiply numerator and denominator by complex conjugate of denominator,

$$C+iS = \frac{2e^{i\theta}}{2-e^{4i\theta}} \times \frac{2-e^{-4i\theta}}{2-e^{-4i\theta}}$$

$$= \frac{4e^{i\theta} - 2e^{-3i\theta}}{4-2e^{-4i\theta}-2e^{4i\theta}+1}$$

$$= \frac{4\cos\theta + 4i\sin\theta - (2\cos 3\theta + 2i\sin(-3\theta))}{4-(2\cos 4\theta + 2i\sin(-4\theta)) - (2\cos 4\theta + 2i\sin 4\theta)+1}$$

$$= \frac{4\cos\theta + 4i\sin\theta - 2\cos 3\theta + 2i\sin 3\theta}{4-2\cos 4\theta + 2i\sin 4\theta - 2\cos 4\theta - 2i\sin 4\theta + 1}$$

$$= \frac{(4\cos\theta - 2\cos 3\theta) + i(4\sin\theta + 2\sin 3\theta)}{5-4\cos 4\theta}$$

$$\therefore S = \mathrm{Im}(C+iS) = \frac{4\sin\theta + 2\sin 3\theta}{5-4\cos 4\theta}$$ (as required)

(Total for Question 4 is 8 marks)

P 6 1 1 7 8 A 0 1 1 2 8

Turn over

5. An engineer is investigating the motion of a sprung diving board at a swimming pool.
Let E be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.
A diver jumps from the diving board.
The vertical displacement, h cm, of the end of the diving board above E is modelled by the differential equation

$$4\frac{d^2h}{dt^2} + 4\frac{dh}{dt} + 37h = 0$$

where t seconds is the time after the diver jumps.

(a) Find a general solution of the differential equation.
(2)

When $t = 0$, the end of the diving board is 20 cm below E and is moving upwards with a speed of 55 cm s^{-1}.

(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above E.
(8)

(c) Comment on the suitability of the model for large values of t.
(2)

(a) Auxilliary Equation: $4m^2 + 4m + 37 = 0$

$$m = \frac{-4 \pm \sqrt{4^2 - 4\times4\times37}}{2\times4}$$

$$= -\frac{1}{2} \pm 3i$$

$$\Rightarrow h(t) = e^{-\frac{1}{2}t}(A\cos 3t + B\sin 3t)$$

(b) Initial Conditions: $h(0) = -20$

$\dot{h}(0) = 55$

Considering the general solution:

$h(0) = e^0(A\cos 0 + B\sin 0) = -20$

$\Rightarrow A = -20$

DO NOT WRITE IN THIS AREA

Question 5 continued

$\dot{h}(t) = -\frac{1}{2}e^{-\frac{1}{2}t}(A\cos 3t + B\sin 3t)$

$+ e^{-\frac{1}{2}t}(-3A\sin 3t + 3B\cos 3t)$

$= -\frac{1}{2}h(t) + e^{-\frac{1}{2}t}(-3A\sin 3t + 3B\cos 3t)$

$\dot{h}(0) = -\frac{1}{2}(-20) + e^{0}(-3A\sin 0 + 3B\cos 0) = 55$

$3B = 45$

$\Rightarrow B = 15$

$\therefore$ For these initial conditions,

$h(t) = e^{-\frac{1}{2}t}(-20\cos 3t + 15\sin 3t)$

At highest point, board is instantaneously at rest:

$\dot{h}(t) = -\frac{1}{2}e^{-\frac{1}{2}t}(-20\cos 3t + 15\sin 3t) + e^{-\frac{1}{2}t}(60\sin 3t + 45\cos 3t) = 0$

$10\cos 3t - \frac{15}{2}\sin 3t + 60\sin 3t + 45\cos 3t = 0$

$55\cos 3t + \frac{105}{2}\sin 3t = 0$

Using R- formulae to combine sine and cosine into a single sine function

$= R\sin(\theta + \alpha) = 0$

Where $R = \sqrt{55^2 + \left(\frac{105}{2}\right)^2} = \frac{25\sqrt{37}}{2}$,

$\alpha = \arctan\left(\frac{105/2}{55}\right) = \arctan\left(\frac{22}{21}\right)$

Question 5 continued

$$\therefore \quad \frac{25\sqrt{37}}{2} \sin\left(3t + \arctan\left(\frac{22}{21}\right)\right) = 0$$

This first happens when argument of sin() is 0

$$3t + \arctan\left(\frac{22}{21}\right) = 0$$

$$\Rightarrow \tan(3t) = -\frac{22}{21}$$

$$3t = \arctan(-22/21) + n\pi \quad , n \in \mathbb{Z}$$

Since $t > 0$, and RHS is first positive for $n = 1$,

$$t = \frac{\arctan\left(-\frac{22}{21}\right) + \pi}{3} = 0.7776476225\ldots$$

To find vertical displacement at this time,

$$h(0.77647\ldots) = e^{-0.5 \times 0.7776\ldots}\left(15\sin(3 \times 0.7776\ldots) - 20\cos(3 \times 0.7776\ldots)\right)$$

$$= 16.71576902\ldots$$

$$\therefore h \approx 16.7 \quad (\text{to } 3 \text{ sf})$$

The maximum vertical displacement of the board is 16.7 cm

c) h decays exponentially with time and this is correct as the displacement of the end should get smaller and smaller. However h never stays still at 0 for large t (the model oscillates indefinitely).

Question 5 continued

(Total for Question 5 is 12 marks)

6. In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2\mathrm{i}$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact.

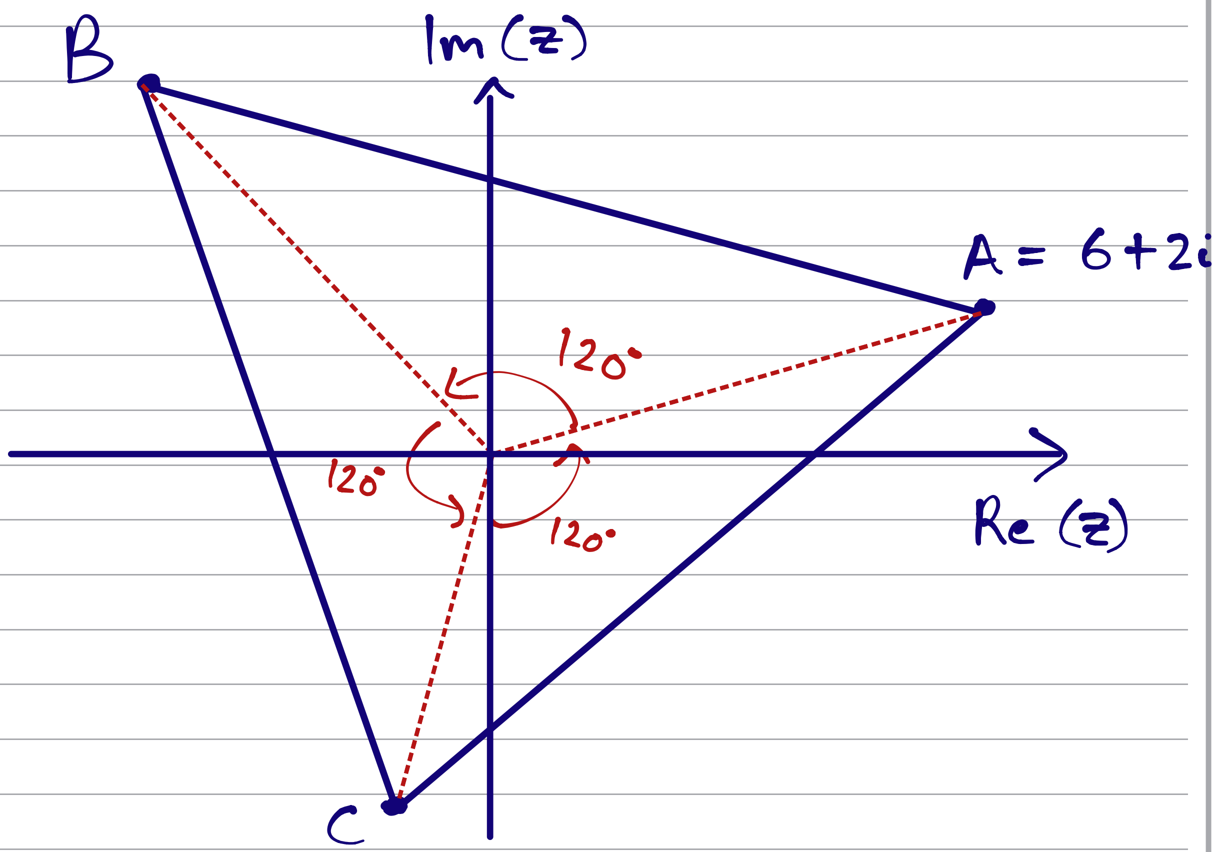
(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

(a)



Rotating the position vector of A 120° about the origin maps A to B

Using a 2D rotation matrix about the origin,

Position Vector of B $= \begin{pmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 6 continued

$$= \begin{pmatrix} -3 - \sqrt{3} \\ 3\sqrt{3} - 1 \end{pmatrix} = \begin{pmatrix} \text{Re}(B) \\ \text{Im}(B) \end{pmatrix}$$

$$\therefore B = \left(-3-\sqrt{3}\right) + \left(3\sqrt{3} - 1\right)i$$

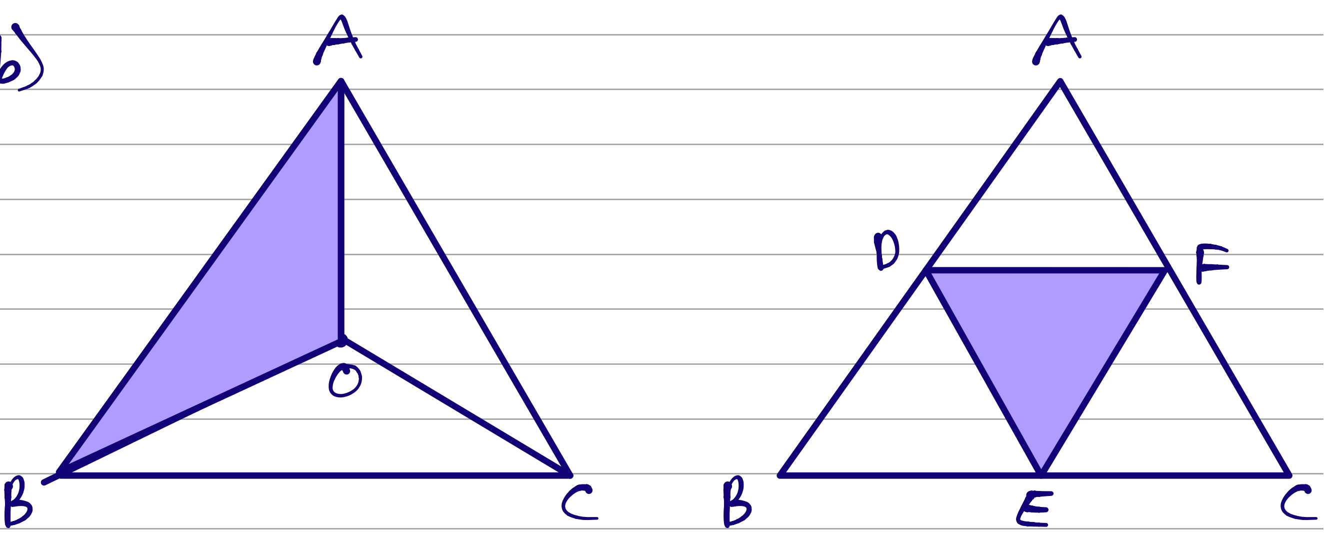
Position Vector of $C = \begin{pmatrix} \cos 240^\circ & -\sin 240^\circ \\ \sin 240^\circ & \cos 240^\circ \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$$= \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 + \sqrt{3} \\ -3\sqrt{3} - 1 \end{pmatrix} = \begin{pmatrix} \text{Re}(C) \\ \text{Im}(C) \end{pmatrix}$$

$$\therefore C = \left(-3+\sqrt{3}\right) + \left(-3\sqrt{3} - 1\right)i$$

(b)



Question 6 continued

Area of $\triangle AOB = \frac{1}{2} \times OA \times OB \times \sin \angle AOB$

$= \frac{1}{2} \times \sqrt{6^2+2^2} \times \sqrt{6^2+2^2} \times \sin 120°$

$= 10\sqrt{3}$

Area of $\triangle ABC = 3 \times$ Area of $\triangle AOB$

$= 30\sqrt{3}$

Area of $\triangle DEF = \frac{1}{4} \times$ Area of $\triangle ABC$

$= \frac{1}{4} \times 30\sqrt{3} = \frac{15\sqrt{3}}{2}$ unit^2

Question 6 continued

(Total for Question 6 is 9 marks)

7.

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Find the values of k for which the matrix $\mathbf{M}$ has an inverse.

(2)

(b) Find, in terms of p, the coordinates of the point where the following planes intersect

$$2x - y + z = p$$
$$3x - 6y + 4z = 1$$
$$3x + 2y - z = 0$$

(5)

(c) (i) Find the value of q for which the set of simultaneous equations

$$2x - y + z = 1 \longrightarrow 1$$
$$3x - 5y + 4z = q \longrightarrow 2$$
$$3x + 2y - z = 0 \longrightarrow 3$$

can be solved.

(ii) For this value of q, interpret the solution of the set of simultaneous equations geometrically.

(4)

(a) M has an inverse $\Leftrightarrow$ det M $\neq 0$

When det M $= 0$,

$$2\begin{vmatrix} k & 4 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} + \begin{vmatrix} 3 & k \\ 3 & 2 \end{vmatrix} = 0$$

$$2(-k-8) + (-3-12) + (6-3k) = 0$$

$$-5k - 25 = 0$$

$$k = -5$$

$\therefore$ M has an inverse when $k \neq -5$

Question 7 continued

(b) Expressing the system of equations in matrix form:

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & -6 & 4 \\ 3 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

The 3×3 matrix is just M with $k = -6$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = M^{-1}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

$M^{-1} = \dfrac{1}{\det M} C^T$ Where C is the matrix of cofactors for elements in A

$$= \frac{1}{-5k-25}\begin{pmatrix} \begin{vmatrix} -6 & 4 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 3 & -6 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ -6 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & -6 \end{vmatrix} \end{pmatrix}^T$$

$$M^{-1} = \frac{1}{5}\begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}$$

Question 7 continued

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} -2p+1 \\ 15p-5 \\ 24p-7 \end{pmatrix}$$

$\therefore$ The planes intersect at $\left(\frac{-2p+1}{5}, 3p-1, \frac{24p-7}{5}\right)$

(c) (i) Since $k=-5$ for this M, M is singular and the 3 planes will NOT intersect at a single point.

BUT, the system can still be consistent. By eliminating z and comparing:

$2 - 4 \times 1$: $5x + y = 4 - q \longrightarrow 4$

$2 + 4 \times 3$: $15x + 3y = q \longrightarrow 5$

Comparing LHS of equations 4 and 5, for the system to be consistent:

$$5 = 3 \times 4$$

$$\Rightarrow q = 3(4-q)$$

$$q = 3$$

Question 7 continued

(ii) Since the system is consistent, there must be infinitely many solutions

∴ The 3 planes form a sheaf

could also be ...

⇒ The 3 planes intersect in a line

* However the 3 planes are not multiples of eachother and ∴ are not parallel with eachother. This concludes that the 3 planes do not intersect in a line *

(Total for Question 7 is 11 marks)

8.

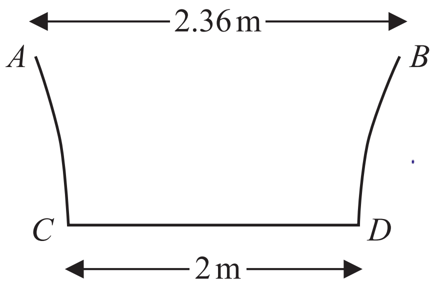


Figure 1

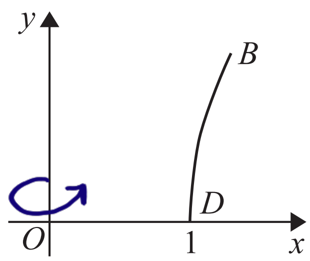


Figure 2

Figure 1 shows the central vertical cross section $ABCD$ of a paddling pool that has a circular horizontal cross section. Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

Using these measurements, the curve BD is modelled by the equation

$$y = \ln(3.6x - k) \qquad 1 \leqslant x \leqslant 1.18$$

as shown in Figure 2.

(a) Find the value of k.

(1)

(b) Find the depth of the paddling pool according to this model.

(2)

The pool is being filled with water from a tap.

(c) Find, in terms of h, the volume of water in the pool when the pool is filled to a depth of h m.

(5)

Given that the pool is being filled at a constant rate of 15 litres every minute,

(d) find, in $\text{cm}\,\text{h}^{-1}$, the rate at which the water level is rising in the pool when the depth of the water is 0.2 m.

(3)

(a) Since BD passes through $(1, 0)$,

$0 = \ln(3.6 - k)$

$\Rightarrow 3.6 - k = e^0 = 1$

$k = \underline{\underline{2.6}}$

P 6 1 1 7 8 A 0 2 4 2 8

Question 8 continued

(b) x - coordinate of B $= \frac{2.36}{2} = 1.18$

depth $= y(1.18) = \ln(3.6 \times 1.18 - 2.6)$

$= 0.4995\ldots$ m

≈ 0.50 m (To 2. sf)

(c) $y = \ln(3.6x - 2.6) \Rightarrow x = \frac{e^y + 2.6}{3.6}$

Using volume of revolution of BD about y axis

Volume of water $= \pi \int_0^h x^2 \, dy$

$= \pi \int_0^h \left(\frac{e^y + 2.6}{3.6}\right)^2 dy$

$= \frac{\pi}{3.6^2} \int_0^h e^{2y} + 5.2e^y + 6.76 \, dy$

$= \frac{\pi}{3.6^2} \left[\frac{1}{2}e^{2y} + 5.2e^y + 6.76y\right]_0^h$

$= \frac{\pi}{3.6^2}\left[\left(\frac{1}{2}e^{2h} + 5.2e^h + 6.76h\right) - \left(\frac{1}{2}e^0 + 5.2e^0 + 0\right)\right]$

$= \frac{\pi}{3.6^2}\left(\frac{1}{2}e^{2h} + 5.2e^h + 6.76h - 5.7\right)$

(d) $\frac{dV}{dh} = \frac{\pi}{3.6^2}\left(e^{2h} + 5.2e^h + 6.76\right)$

Question 8 continued

$$\left.\frac{dV}{dh}\right|_{h=0.2} = \frac{\pi}{3.6^2}\left(e^{0.4} + 5.2e^{0.2} + 6.76\right) = 3.539\ldots$$

By the chain rule,

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=0.2} = \frac{1}{\left.\frac{dV}{dh}\right|_{h=0.2}} \times \frac{dV}{dt} \qquad \frac{dV}{dt} = \frac{15}{1000} \times 60$$

$$= \frac{1}{3.539\ldots} \times 0.015 \times 60$$

$$= \underline{\underline{25.4}} \text{ cm per hour}$$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS